光学 精密工程
Optics and Precision Engineering

文章编号　1004-924X(2007)04-0604-07

调制频率抖动对相干检测的影响及消除算法

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摘要:介绍了调制频率偏移抖动的起因,并分析了相干检测过程中由于调制频率偏移抖动所引起的误差,对模拟和数字相干调制和解调过程建立了数学模型。基于文中所建立的数学模型,确定了调制频率偏移抖动对模拟和数字相干调制和解调过程的影响,特别是其对离散采样过程的影响关系。最后,提出了一种基于模糊逻辑的隶属函数加权采样算法,并进行了实验。实验结果表明,使用隶属函数加权采样算法后,当调制频率偏移范围小于±9.2%时,相干检测的精度未受影响;而未使用隶属函数加权采样算法时,当调制频率偏移范围大于±3%时,相干检测的精度便会受到影响。结果还表明,调制频率偏移抖动对模拟和数字相干检测过程影响的数学模型在相干检测应用领域很有价值,提出的隶属函数加权采样算法能够有效地消除调制频率偏移对相干检测的影响,其效果已经在应用中得到了验证。
关　键　词:相干检测;本征频率;隶属函数;模糊逻辑
中图分类号:O436.1　　**文献标识码:**A

Effect of modulation frequency Jitter on coherent detection and its elimination algorithm

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Abstract: The causation of MFJE (Modulation Frequency Jitter and Excursion) is generally introduced, and the error of MFJE in coherent detection is analyzed at the same time, the mathematic models of MFJE error in analog and digital coherent detection process are built. Based on the mathematic models, the effects of MFJE on analog and digital coherent modulation-demodulation process are identified, especially the effect of MFJE on discrete sampling. Finally, a Membership Weighted Function Sampling (MWFS) algorithm is proposed. The experimental result shows that the precision of coherent detection has not been affected in a modulation frequency excursion range of ±9.2% with MWFS, compared with the range of ±3% without MWFS. The result also shows that the mathematic models of MFJE error in analog and digital coherent detection process are valuable in coherent detection application field and MWFS proposed in the paper can eliminate the effect of modulation frequency excursion on coherent detection efficiently, which has been certified in practice.
Key words: coherent detection; latent frequency; membership function; fuzzy set

收稿日期:2006-12-22;**修订日期:**2007-02-18.
基金项目:国家高技术研究发展计划(863)资助课题

1 Introduction

Coherent detection is widely used in the field of weak signal detection. In the process of detection, modulation signal is rebuilt as the reference signal, which should have the same frequency with modulation signal for better demodulation precision. Especially in application of Optical Fiber Coherent Detection (OFCD) based on coherent detection require that the frequency of Modulation Square Wave (MSW) is consistent with the Optical Fiber Loop Interference Latent Frequency (OFLILF)[1,4]. But the excursion and jitter of modulation frequency exist in practice. These analyses are very valuable for improving the effect in Fiber Optical Gyroscope (FOG)[2,3] and other applications. Herve C. Lefevre has made a brief analysis in paper[1], but has neither made quantification for this effect; nor provided the method of compensating the effect. A. G. burr has described the phase noise in coherent modulation in paper[5], he has presented the effect on modulation frequency stability. Pierre-Alain Nicati has presented the frequency stability of a Brillouin fiber ring laser in paper[6].

Error effect of MFJE on coherent detection is analyzed and the mathematic models of MFJE error in analog and digital coherent detection process are built in this paper. Through the model, effects of MFJE on analog and digital coherent modulation-demodulation process are identified, especially the effect of MFJE on discrete sampling. A MWFS algorithm based on fuzzy set is proposed at the end of paper. Coherent demodulation error of MFJE can be eliminated efficiently through selecting the appropriate weighted function, which has been certified in simulations and experiments. Mathematic models in this paper, which are very useful in coherent detection study, can be applied in the field of coherent detection process simulation, signal process principle certification and noise analysis. The MWFS algorithm based on fuzzy set can be used in many fields of high speed sampling and high speed signal proceeding, and has more merits than mean sampling algorithm.

2 Reason of frequency jitter

Strict consistency between MSW frequency ω and interference Latent Frequency (ILF) ω_R is required in Coherent Light Detection (CLD) [7,8], but it is hard to assure that in practice, because of the restriction of noise and crystal oscillator stability. The frequency ω always jitters at a range of $\Delta\omega$ near by frequency.

At prior time, D. B. Leeson has proposed a simple model of oscillator noise in paper [9]. And later, the deeper analysis has been presented in paper [10] and [11], where the authors have compared their models with Leeson's model.

3 MFJE on analog and digital coherence detection

With the feature of signal's periodicity and noise's randomization, coherent detection algorithm filters off noise using cross correlation function. Fiber Optical Gyroscope (FOG) is one of the important applications of CLD, which was presented in paper[1]. Supposing the frequency difference between MSW frequency and OFLILF ω_R is $\Delta\omega$ in coherent detection algorithm. Equation of MSW is shown as:

$$D(t)=\frac{1}{2}+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos[(2n+1)(\omega_R+\Delta\omega)t], \quad (1)$$

In the process of modulation, MSW is used to modulate the phase of Clockwise (CW) and Counter Clockwise (CCW) transmit light signal, which can be expressed as follows:

$$V_i(t)=V_O\{1+\cos[\Delta\phi_R+\varphi_S S(t)]\}, \quad (2)$$

Where $S(t)=D(t)-D(t+\frac{T}{2})$, $\Delta\varphi_R$ is the

SAGNAC frequency difference of fiber optical loop's turning and $\Delta\varphi_R=\frac{2\pi LD}{\lambda C}\Omega$, φ_S can be chosen as $\frac{\pi}{2}$ and V_O is amplitude of input signal.

During demodulation, reference signal $V_r(t)$ and the detected signal have the same frequency produced by the same signal source. Using the Zero Phase Difference Detection (ZPDD), the equation is defined as follows:

$$\begin{cases} V_r(t)=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\cos[(2n+1)(\omega_R+\Delta\omega)t] \\ V_i(t)=V_O\{1+\cos[\Delta\varphi_R+\varphi_S\cdot S(t)]\} \\ V_i(t)V_r(t-\tau)=V_OV_r(t-\tau)\times\{1+ \\ \qquad \cos\Delta\varphi_R\cos[\frac{\pi}{2}S(t)]- \\ \qquad \sin\Delta\varphi_R\sin[\frac{\pi}{2}S(t)]\} \\ V_{out}=\int_{-\frac{T}{2}}^{\frac{T}{2}}V_i(t)V_r(t-\tau)\mathrm{d}t \quad (\tau=0) \end{cases}, \quad (3)$$

Where V_{out} is the output signal of coherent detection, and τ is coherent delay time.

Low Pass Filter (LPF) is commonly used with upper limit cutoff frequency on $1/(2\Delta\tau_t)=\omega_R$ to replace integrator in coherent detection circuit design. After the processing of LPF, the signal $\omega>\omega_R$ in equation (3) will be filtered off, when $\omega=\omega_R+\Delta\omega$. Substitute the parameter of equation (3) into coherent demodulation expression, which is shown as follows:

$$\begin{aligned} V_{out}=&\int_{-\frac{T}{2}}^{\frac{T}{2}}V_i(t)V_r(t-\tau)\mathrm{d}t= \\ &LPF[V_i(t)V_r(t-\tau)]= \\ &\frac{2V_O}{\pi^2}\{(\cos\Delta\phi_R-\sin\Delta\phi_R)\times \\ &[\cos(\pi+\frac{\Delta\omega\cdot T_R}{2})+\sin(\pi+\frac{\Delta\omega\cdot T_R}{2})]\}, \end{aligned} \quad (4)$$

If modulation frequency is equal to the OFLILF, that is $\omega=\omega_R$, $\Delta\omega=0$, and $V_{out}=-\frac{2V}{\pi^2}(\sin\Delta\phi_R-\cos\Delta\phi_R)$. Otherwise, if modulation frequency is not equal to the OFLILF, the error factor $\cos(\pi+\frac{\Delta\omega\cdot T_R}{2})+\sin(\pi+\frac{\Delta\omega\cdot T_R}{2})$ will exist in V_{out}, which will affect the result of coherent detection. Error factor affects the open loop output in applications, and furthermore, it will make an effect on bias instability of FOG.

Digital Coherent Detection (DCD) is based on discrete signal. Selection of sampling point has a great effect on the result of demodulation because input signal $V_i(t)$ is sampled and converted to discrete signal $V_i(n)$ before DCD process.

Reference signal $V_S(t)$ is square wave which has the same frequency and phase with input signal $V_i(t)$. Suppose the sampling point number in half period of $V_S(t)$ is N, digital demodulation equation can be shown as follows:

$$\begin{aligned} V_{out}=&\frac{1}{2N}\sum_{n=1}^{2N}V_i(n)V_r(n)= \\ &\frac{1}{2N}[\sum_{n=1}^{N}V_i^+(n)-\sum_{n=N+1}^{2N}V_i^-(n)], \end{aligned} \quad (5)$$

Based on equation (5), difference which remains after the sampling value in positive half period subtracted from same phase one in negative half period can be accumulated to get the coherent detection output V_{out}. Excursion of modulation frequency will equally affect the input signal and reference signal. Suppose modulation frequency ω has been converted as $\omega=\omega_R-\Delta\omega$, the excursion of it makes the sampling number of input signal $V_i(t)$ in half period changing to $N-\Delta N$. Discrete signal $V_i(n)$ sampled from input signal $V_i(t)$ can be defined as:

$$\begin{cases} \sum_{n=1}^{\infty}V_i(n)=V_O(1+\sum_{n=1}^{\Delta N}\cos\Delta\phi_R 0\sum_{n=\Delta N+1}^{N-\Delta N}\sin\Delta\phi_R+ \\ \qquad \sum_{n=N-\Delta N+1}^{N}\cos\Delta\phi_R+\sum_{n=N+1}^{2(N-\Delta N)}\sin\Delta\phi_R+ \\ \qquad \sum_{n=2(N-\Delta N)+1}^{2N-\Delta N}\cos\Delta\phi_R+\cdots\cdots) \\ \sum_{n=1}^{\Delta N}V_i^+(n)=\sum_{n=N-\Delta N+1}^{N}V_i^-(n) \end{cases}, \quad (6)$$

Expression of coherent demodulation in

MFJE can be deduced as follows:

$$V_{out} = \frac{1}{2(N-\Delta N)}\left[\sum_{n=\Delta N+1}^{N-\Delta N} V_i^+(n) - \sum_{n=N+1}^{2(N-\Delta N)} V_i^-(n)\right], \quad (7)$$

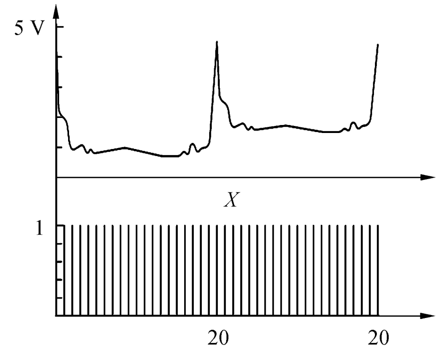
Based on equation (5), the effective sampling point number of coherent detection output V_{out} is $N-2\Delta N$, and there are ΔN of sampling points which are ineffective and eliminated each other. After the mean operation, the output signal V_{out} will be affected by the error factor $(N-2\Delta N)/(N-\Delta N)$, and warp from really coherent detection result. In many applications of DCD, many error and excursion are related with the effect above. For instance, bios jitter and dead band has some relations in digital closed loop FOG.

4 Effect eliminating algorithm

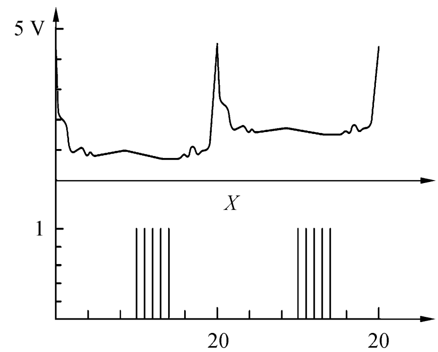
Analysis upwards and simulations show that MFJE, which causes little and mainly high order effect on the middle part of sampling period, focus the effect on the points at two ends of sampling period[12]. Membership function algorithm based on fuzzy set can reduce or filter off the weights of sampling points at two sides, which affect DCD mostly. Suppose all the sampling points belong to set A, membership function of A is $\mu_n(A)$, and $\mu_n(A)\in[0,1]$, and membership function vector is $\boldsymbol{\mu}(A)=[\mu_1(A), \mu_2(A), \cdots \mu_N(A)]$. Weighting the sampling value by properly setting the value of $\boldsymbol{\mu}(A)$, large at middle part and little at two ends, can get to the purpose[13,14]. The equation of weighting sampling points by $\boldsymbol{\mu}(A)$ can be defined as follows:

$$V_{out} = \left[\sum_{n=1}^{N-\Delta N} \mu_n(A)V_s^+(n) - \sum_{n=N-\Delta N+1}^{2(N-\Delta N)} \mu_n(A)V_s^-(n)\right] \Big/ \left(\sum_{n=1}^{2(N-\Delta N)} \mu_n(A)\right), \quad (8)$$

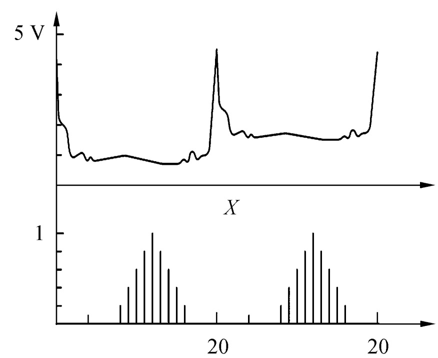
There are three sampling weighted algorithms with membership function in Fig 1, (a) shows a mean sampling, (b) show a rectangle weighted sampling and (c) shows a delta weighted sampling.



(a) Mean sampling



(b) Rectangle weighted sampling

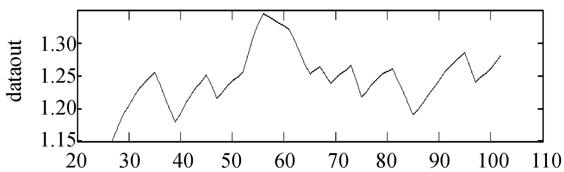


(c) Delta weighted sampling

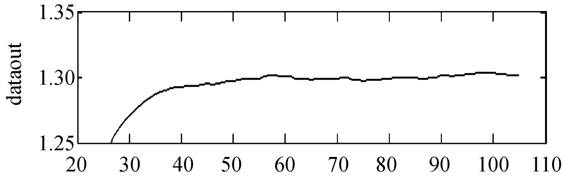
Fig. 1 Three kinds of sampling weighted algorithm

5 Simulations and experiments

Simulating the MFJE of coherent detection in MATLAB and assuming the jitter of modulation frequency accord with sin function in a range of 5%. Output data have an acute jitter when using mean sampling algorithm, which is shown in Fig 2(a). If input data are weighted by an appropriate membership function, effect on modulation frequency excursion can be filtered off and the output can be more accurate, which is shown in Fig(b).



(a) Result of mean sampling

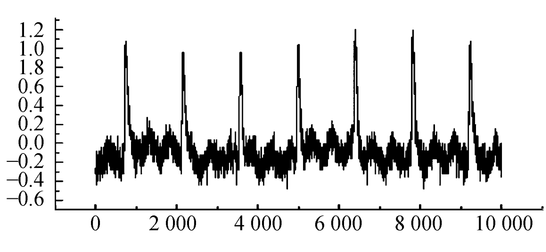


(b) Result of weighted sampling

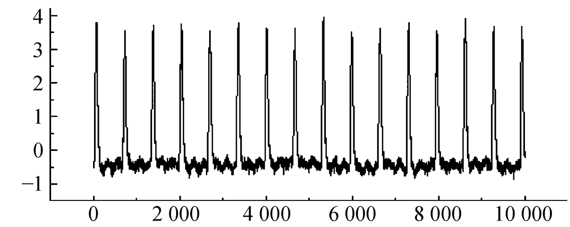
Fig. 2 Sampling results

OFLILF is 176 kHz in experiment, when modulation frequency is 176 kHz, the input phase modulation waveform is shown in Fig. 3 (a), and when modulation frequency is 190 kHz, the input phase modulation waveform is shown in Fig. 3(b).

Weighting the sampling points with rectangle membership function, the sampling equation can be shown as:



(a) Phase modulation waveform in 176 kHz modulation frequency



(b) Phase modulation waveform in 190 kHz modulation frequency

Fig. 3 Phase modulation waveform in 176 kHz and 190 kHz

$$\begin{cases} V_{out} = \left[\sum_{n=1}^{N-\Delta N} p_n V_i^+(n) - \sum_{n=N-\Delta N+1}^{2(N-\Delta N)} p_n V_i^-(n)\right] / \left(\sum_{n=1}^{2(N-\Delta N)} p_n\right) \\ p_n = 1 \quad [15 \leqslant n \leqslant 22], \end{cases} \tag{9}$$

Experiment result shows that coherent detection has much less requirement of the frequency stability when using weighted sampling algorithm. Excursion of modulation frequency doesn't affect the coherent detection obviously, and output data doesn't change acutely when modulation frequency excursion exists.

6 Conclusion

The MFJE exists in the process of coherent light modulation and demodulation practically. In this paper, effect on digital and analog coherent demodulation process is analyzed and mathematic models of MFJE error in analog and digital coherent detection process are built. Furthermore, simulation and experiments based on FOG of MFJE in coherent detection process has been shown in this paper. An MFJE effect eliminating algorithm based on membership function in fuzzy set has been proposed, and it has been certified in experiments that proper membership weighted sampling algorithm can reduce the effect of modulation frequency excursion efficiently. Experiment result shows that the precision of coherent detection has not been affected in a modulation frequency excursion range of ±9.2% with MWFS, compared with the range of ±3% without MWFS.

Weighting function proposed in this paper is fixed, which has an efficiently function when modulation frequency is excursion major and jitter minor. A feedback error factor in demodulation process will be imported in further research and function will be adjusted by feedback error factor to be self adapting. Error of MFJE will be eliminated more efficiently. There are many forms of modulation frequency jitter and effects of modulation and demodulation process are difference in different jitter form. More detailed analysis in this issue will be made in further research.

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